

# Microlensing pulsars

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## ABSTRACT

We investigate the possibilities that pulsars act as the lens in gravitational microlensing events towards the galactic bulge or a spiral arm. Our estimation is based on expectant survey and observations of FAST (Five hundred meter Aperture Spherical Telescope) and SKA (Square Kilometer Array), and two different models of pulsar distribution are used. We find that the lensing rate is  $\geq 1$  event/decade, being high enough to search the real events. Therefore, the microlensing observations focusing on pulsars identified by FAST or SKA in the future are meaningful. As an independent determination of pulsar mass, a future detection of microlensing pulsars should be significant in the history of studying pulsars, especially in constraining the state of matter (either hadronic or quark matter) at supra-nuclear densities. The observations of such events by using advanced optical facilities (e.g., the James Webb Space Telescope and the Thirty Meter Telescope) in future are highly suggested.

**Key words:** gravitational lensing - pulsars: general - stars: neutron.

## 1 INTRODUCTION

Pulsars could be normal neutron stars or quark stars (Lattimer & Prakash 2004), and it should be very important to affirm or negate the existence of either neutron or quark stars in order to guide physicists in studying the nature of fundamental strong interaction. With regard to the possible ways of identifying quark stars (e.g., Xu 2008, for a review), it could be very straightforward and clear if we would find low-mass quark stars, since neutron stars are essentially gravitation-bound while low-mass quark stars are mainly confined by strong interaction. If we can detect a pulsar-like star with mass  $\lesssim 0.1M_{\odot}$ , then it is surely a quark star.

How to measure the mass of stars? Up to now only the masses of compact stars in binary systems have been determined, by either Keplerian or post-Keplerian parameters. The lowest mass detected of an eclipsing X-ray pulsar, SMC X-1, is  $1.06^{+0.11}_{-0.10} M_{\odot}$ , which is near the minimum mass expected for a neutron star produced in a supernova (van der Meer et al. 2007). However, most of pulsars are isolated, and it is still a big challenge to measure their masses. It was suggested to observationally determine an isolated neutron star's mass from the red-shift (as a function of the ratio of mass to radius,  $M/R$ ) and pressure broadening (as a function of  $M/R^2$ ) of an absorption spectrum, yet no atomic line has been detected with certainty in the thermal X-ray spectra (e.g., Xu 2002).

Interestingly, gravitational microlensing (e.g., Mao

2008), which makes use of the temporal brightening of a background star due to intervening object, would provide us a powerful method to measure the masses of compact objects. The amplification  $A$  of a background star by the passage of a pulsar has a relation of

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}},$$

where  $u = r'/R_E$  is a parameter, with the undeflected distance,  $r'$ , and the Einstein radius,  $R_E$  (Paczynski 1986), which is almost a function of pulsar mass  $M$  and  $r$  if the background star's distance  $D \gg r$  [see Eq.(1)]. One can then directly obtain the mass-distribution of the lens objects via the light curve (Glicenstein 2003). With the method of microlensing, we can not only determine the mass distribution of massive compact halo objects (MACHO, Alcock et al. 1997) and massive objects in the Galactic center region (Wex et al. 1996), but may also measure the masses of isolated pulsars.

The possibilities of microlensing neutron star have been discussed previously and several estimation of lensing rates had been presented in the literatures (Horvath 1996; Schwarz & Seidel 2002). In this paper we reestimate the lensing rate by calculating the solid angle Einstein ring swept due to pulsar's proper motion, using two different pulsar distribution models. In addition, our estimation also takes expectant observations of FAST (Nan et al. 2006) and SKA (Johnston et al. 2007) into account, since the two new telescopes, FAST and SKA, will greatly enhance our

ability of searching and observing pulsars, and the number of pulsars with measured proper motion in the Galaxy is crucial to estimate the lensing rate. Lorimer et al. (2006) used the results from recent surveys with the Parkes Multi-beam system to derive a potentially detectable population of 30000 normal pulsars. By using this number and assuming 30000 potentially detectable millisecond pulsars in the Galaxy (Lyne et al. 1998), Smits et al. (2009a) estimated that the all-sky survey with only the 1-km core of the SKA located in the southern hemispheric would detect about 14000 normal pulsars and about 6000 millisecond pulsars. Under the same assumption, Smits et al. (2009b) noted that more than 7000 previously unknown pulsars could be discovered by FAST in the Galactic plane ( $|b| < 10^\circ$ ) with an observation time of 1800 s per-beam. Further more, by regular timing these pulsars using FAST and SKA, the accurate positions and proper motion of them can also be determined. We then propose to search possible microlensing events due to pulsars identified by FAST and SKA and to carry out microlensing observations coupled with radio observations by means of future optical projects (e.g., the James Webb Space Telescope<sup>1</sup> or the Thirty Meter Telescope Project<sup>2</sup>).

The lensing rate of our result indicates a high possibility of observing microlensing event due to pulsars in the future with the help of FAST and SKA. We expect an independent measurement of pulsar mass through these microlensing observations, especially to discovery low-mass pulsars, in order to understand the real nature of pulsars.

## 2 THE LENSING RATE

We consider a pulsar at distance  $r$  with velocity  $v$ , microlensing a background star at distance  $D$ . The Einstein ring of this pulsar sweeps a solid angle  $S_N$  on the celestial sphere during a period time of  $t$ . The Einstein radius is

$$R_E = \sqrt{\frac{2R_S}{D}} r(D-r), \quad (1)$$

where  $R_S = 2GM/c^2$  denotes the Schwarzschild radius. This solid angle  $S_N$  depends on the mass  $M$ , the distance  $r$  to the pulsar and also the distance  $D$  to the star,

$$S_N(M, v, r, D) = \frac{R_E}{r} \frac{vt}{r}. \quad (2)$$

We expect that a microlensing event occurs when a star in the background falls into this solid angle. We assume that our telescopes can identify a total of  $N$  pulsars, and  $S$  square degrees of the Milky Way is visible to it, then the lensing rate per unit time can be estimated as

$$p = \frac{\sum S_N}{S} N_{\text{star}}, \quad (3)$$

where  $N_{\text{star}}$  denotes the total number of stars visible.

If we take pulsar distribution into account, the sum turns into multiple integral on the space of position ( $r$  and  $D$ ) and velocity ( $\vec{v}$ ). Then the lensing rate  $p$  can be rewritten as

$$p = \frac{\int_{\text{position}} \int_{\text{velocity}} S_N(\vec{v}, \vec{r}, \vec{D}) d\vec{v} d\vec{r} d\vec{D}}{S} N_{\text{star}}, \quad (4)$$

where  $D \geq r$ . We assume that the number density of stars in the galactic bulge or a spiral arm is constant in following estimation. Because the velocity distribution of pulsars is not a function of  $r$  and  $D$ , one can integrate the velocity distribution independently of position distribution. We can thus just apply a mean velocity in estimating the lensing rate. Hansen and Phinney (1997) suggested a Maxwellian distribution for the kick velocities of pulsars,

$$f(v_{\text{kick}}) = \sqrt{\frac{2}{\pi}} \frac{v_{\text{kick}}^2}{\sigma^3} \exp(-\frac{v_{\text{kick}}^2}{2\sigma^2}), \quad (5)$$

where  $\sigma = 190$  km/s and mean kick velocity  $v_{\text{kick}} = 300$  km/s are chosen. Considering that only the part of the velocity vector that lies in the lens plane is effective, one should use the mean velocity projection on the lens plane (Schwarz & Seidel 2002),

$$v = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} v_{\text{kick}} \cos \alpha d\alpha, \quad (6)$$

where the projected velocity,  $v$ , is order of 200 km/s.

We use two different models of pulsar number density from Hartman et al. (1997), which resemble the models from Narayan (1987) and Johnston (1994), see also Schwarz and Seidel (2002), in our following simulations. The first model of pulsar distribution is

$$n_{P1}(R) = \frac{1}{2\pi R_W^2} \exp(-\frac{R}{R_W}), \quad (7)$$

where  $R$  is the radial distance of the pulsar to the galactic center in the galactic plane, and  $R_W = 5$  kpc. The second model is

$$n_{P2}(R) = \frac{c_{P2}}{2\pi R_W^2} \exp(-\frac{(R - R_{\text{max}})^2}{2R_W^2}), \quad (8)$$

where  $R_W = 1.8$  kpc and  $R_{\text{max}} = 3.5$  kpc. The normalization constant is  $c_{P2} = 0.204$  for the given choice of  $R_{\text{max}}$ . For the  $z$ -dependence we apply

$$n_z(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2} \frac{z^2}{\sigma^2}), \quad (9)$$

with  $\sigma = 0.45$  kpc (Lyne et al. 1998; Schwarz & Seidel 2002).

We transform the pulsar distribution into the spherical coordinates  $(r, \theta, \phi)$  with the sun at the origin via

$$z = r \sin \theta, \quad (10)$$

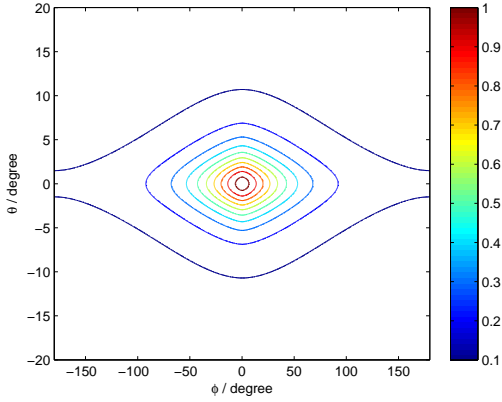
$$R^2 = r^2 \cos^2 \theta + R_{\text{SC}}^2 - 2rR_{\text{SC}} \cos \theta \cos \phi, \quad (11)$$

where  $R$  is the radial distance of the pulsar to the galactic center in the galactic plane and  $z$  is the height, and  $R_{\text{SC}} = 8.5$  kpc. For pulsar distribution  $n_{P1}$ , we have

$$\begin{aligned} p &= \frac{NN_{\text{star}}}{S} \iiint n_{P1}(R) n_z(z) r^2 \sin \theta dr d\theta d\phi \\ &\times \frac{1}{r^2} \frac{vt}{4\pi r} \int_0^r \sqrt{\frac{2R_S}{D}} r(D-r) dD \\ &= \frac{NN_{\text{star}} vt}{2\pi S R_W^2 \sqrt{2\pi}\sigma} \iiint \end{aligned}$$

<sup>1</sup> <http://www.jwst.nasa.gov/>

<sup>2</sup> <http://www.tmt.org/>



**Figure 1.** The lensing rate density as function of  $\theta$  and  $\phi$  in the model of pulsar distribution described by Eq.(7), where  $\theta$  is the azimuth angle and  $\phi$  is the polar angle. Color bar stands for lensing rate in units of number of events/decade/degree<sup>2</sup>.

$$\exp\left(-\frac{\sqrt{r^2 \cos^2 \theta + R_{\text{SC}}^2 - 2rR_{\text{SC}} \cos \theta \cos \phi}}{R_{\text{W}}}\right) \exp\left(-\frac{1}{2} \frac{r^2 \sin^2 \theta}{\sigma^2}\right) \sin \theta dr d\theta d\phi \frac{\sqrt{2R_{\text{S}}r}}{4\pi} \int_0^1 \sqrt{\frac{y-1}{y}} dy, \quad (12)$$

where  $y = D/r$ . If we define  $x = r/R_{\text{SC}}$ , then the expression can be reduced to

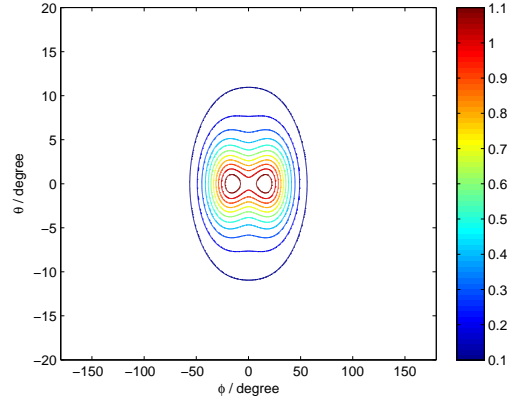
$$p = \frac{NN_{\text{star}}vtR_{\text{SC}}\sqrt{2R_{\text{S}}R_{\text{SC}}}}{8\pi^2SR_{\text{W}}^2\sqrt{2\pi}\sigma} \iint \exp\left(-\frac{\sqrt{x^2 \cos^2 \theta + 1 - 2x \cos \theta \cos \phi}}{R_{\text{W}}/R_{\text{SC}}}\right) \exp\left(-\frac{1}{2} \frac{x^2 \sin^2 \theta R_{\text{SC}}^2}{\sigma^2}\right) \sin \theta dx d\theta d\phi \int_0^1 \sqrt{\frac{y-1}{y}} dy. \quad (13)$$

We consider that FAST is expected to detect about 7770 pulsars in about  $70 \times 10$  square degrees of the Milky Way of observing time in less than a year (Nan et al. 2006), while SKA is expected to detect about 15000 pulsars in  $290 \times 10$  square degrees of the Milky Way. If one chooses  $M = 1M_{\odot}$ ,  $r \in [0, 5\text{kpc}]$ ,  $t = 1$  yr,  $N = 15000$ ,  $N_{\text{star}} = 10^{11}$ ,  $v = 200$  km/s, and  $S = 2000$  square degrees, then we have  $p \approx 12$  event/year from Eq.(13).

Certainly the total number of stars visible is less than  $10^{11}$  if the effect of extinction is included. Based on the photometric maps of the galactic bulge released by the OGLE (Optical Gravitational Lensing Experiment) project, which contain photometry of about 30 million stars from 49 fields covering 11 square degrees in different regions of the galactic bulge (Udalski et al. 2002), we estimate that in the region of  $S = 2000$  square degrees,  $10^9$  stars would be visible, then the lensing rate should be  $p \geq 1$  event/decade according to Eq.(13). The lensing rate density as function of  $\theta$  and  $\phi$  is shown in Fig.1.

For pulsar distribution  $n_{\text{P2}}$  in the model of pulsar distribution described by Eq.(8), we have accordingly

$$p = \frac{NN_{\text{star}}}{S} \iiint n_{\text{P2}}(R)n_z(z)r^2 \sin \theta dr d\theta d\phi \times \frac{1}{r^2} \frac{vt}{4\pi r} \int_0^r \sqrt{\frac{2R_{\text{S}}}{D}} r(D-r) dD$$



**Figure 2.** Same as in Fig. 1, but in the model of pulsar distribution described by Eq.(8).

$$= \frac{NN_{\text{star}}c_{\text{P2}}vt}{2\pi SR_{\text{W}}^2\sqrt{2\pi}\sigma} \iiint \exp\left(-\frac{(\sqrt{r^2 \cos^2 \theta + R_{\text{SC}}^2 - 2rR_{\text{SC}} \cos \theta \cos \phi} - R_{\text{max}})^2}{2R_{\text{W}}^2}\right) \exp\left(-\frac{1}{2} \frac{r^2 \sin^2 \theta}{\sigma^2}\right) \sin \theta dr d\theta d\phi \frac{\sqrt{2R_{\text{S}}r}}{4\pi} \int_0^1 \sqrt{\frac{y-1}{y}} dy. \quad (14)$$

From the same process as for  $n_{\text{P1}}$  of Eq.(7), then we have  $p \geq 2$  event/decade if one chooses  $M = 1M_{\odot}$ ,  $r \in [0, 5\text{kpc}]$ ,  $t = 1$  yr,  $N = 15000$ ,  $N_{\text{star}} = 10^{11}$ ,  $v = 200$  km/s,  $S = 2000$  square degrees, and takes the effect of extinction into account. The corresponding lensing rate density as function of  $\theta$  and  $\phi$  is shown in Fig.2.

The lensing rates estimated above are for the microlensing events in the galactic center. Considering that the observation of FAST is limited to the spatial arms and that pulsars mainly distribute on the disk of the Milky Way, we then transform the pulsar distribution into cylindrical coordinates with sun at the origin via

$$R^2 = r^2 + R_{\text{SC}}^2 - 2rR_{\text{SC}}\cos\phi. \quad (15)$$

Using the cylindrical coordinates, we have

$$p = \frac{NN_{\text{star}}}{S} \iint n(r, \phi)n_z(r)rdr d\phi \times \frac{1}{r^2} \frac{vt}{r} \int_0^r \sqrt{\frac{2R_{\text{S}}}{D}} r(D-r) dD. \quad (16)$$

For pulsar distributions of both  $n_{\text{P1}}$  and  $n_{\text{P2}}$ , we calculate following the way we did previously and choose  $M = 1M_{\odot}$ ,  $r \in [0, 5\text{kpc}]$ ,  $t = 1\text{yr}$ ,  $N = 10000$ ,  $v = 200$  km/s, and  $S = 2000$  square degrees. Considering the effect of extinction, we choose  $N_{\text{star}} = 10^9$ . Finally we can also obtain a lensing probability of  $p \geq 1$  event/decade for FAST.

### 3 A PROPOSAL OF DISCOVERING MICROLENSING PULSARS

We have estimated the probability of observing microlensing pulsars by means of FAST and SKA. Besides the encouraging lensing rate according to our results, a feasible searching

strategy is also essential for discovering a microlensing pulsar. We therefore propose to carry out microlensing pulsar observation coupled with FAST and SKA observations in the future, which are hopeful to determine the mass of isolated pulsars.

Microlensing pulsar observation should be based on the expectant data of FAST and SKA. We propose to systematically list the information of new pulsars discovered by FAST and SKA in the future, including the position, proper motion and distance, and high proper motion pulsars should be especially focused on. The position of these pulsars in the next few decades could also be predicted. Then we could compare the predicted position of these pulsars with the position of stars in the galactic bulge and spiral arms, and background star candidates of microlensing events whose position would be quite close to the predicted position of pulsars could be picked up. With these candidates, we should formulate a plan to monitor certain background stars for the predicted microlensing events in a certain period of time in the future.

Microlensing pulsar observation could be carried out on ongoing lensing projects, and future advanced facilities are also expected to benefit the observation. The magnification of a background star could be about 0.3, on the condition that a pulsar just arriving at the edge of the Einstein ring (Mao 2008), and the time scale of the microlensing event should be about 15 days. The observation can then be done on current telescopes in the infrared band where the interstellar extinction is much weaker (Horvath 1996; Udalski et al. 2002). Besides ongoing gravitational lensing projects, we could also look forward to future projects, especially the Thirty Meter Telescope (TMT) Project. The TMT project, which is scheduled for the next decade, will greatly promote our study of gravitational lensing on cosmology, galaxy formation and the distribution of lensing mass. TMT's great capabilities of resolving smaller, fainter source populations will allow a much higher sky density of background sources to be used (Carlberg 2004), which will directly enhance the possibility of discovering microlensing pulsar events. Therefore, TMT together with FAST and SKA will provide us a good opportunity to observe microlensing pulsar events and determine the mass of isolated pulsars.

We note that the proposed microlensing pulsar observation is costless, that is to say, it does not need to monitor a target all the time. Our estimation has shown the possibility of microlensing a pulsar, and the key of the observation is predicting the microlensing events based on the large and sensitive database of FAST and SKA, and then monitoring microlensing candidates of background stars in a predicted short period of time. Even though the prediction can not be precise and the lensing rate is relatively small, the costless observation is still meaningful and should be carried out.

#### 4 DISCUSSION AND CONCLUSIONS

We have estimated the lensing rates of pulsars towards the galactic bulge and spiral arms. Our estimation shows that, for FAST and SKA, the lensing rates ( $\propto N$ ) are  $\geq 1$  event/decade at least, which are much higher than the estimations made in previous literatures. The number of pulsars with measured proper motion,  $N$ , is crucial to estimate the

rate of lensing events. In table 1, we summarize the expectant number of pulsars to be detected by FAST and SKA, based on previous survey simulation results. In addition, the lensing rate should increase significantly if the population of rotating radio transients (RRATs, Keane 2009) is included, since the total number of RRATs is at least several times of galactic active radio pulsars (McLaughlin 2006). RRATs, which can be precisely located by timing and dispersion measurement, should then be one of the key targets of FAST and SKA.

We also note that the lensing rates we showed are based on photometric microlensing events. If we consider astrometric microlensing, which makes use of the shift of the centroid of the combined images of the light source, the cross section would increase by a factor of  $\sim 10^2$  (Horvath 1996; Schwarz & Seidel 2002). As our lensing rate is proportional to cross section, the astrometric microlensing probability would be  $\sim 10^2$  times higher than that presented previously in §2. It could then be realistic to search astrometric microlensing pulsars and to detect the events by facilities with very high position precision in the future.

The lensing rates indicate that it is hopeful to measure the mass of an isolated pulsar in the future with the method of microlensing. We propose to do catalogue comparison and microlensing prediction for pulsars identified by SKA and FAST in the future, and to carry out microlensing observation coupled with radio observation in order to detect microlensing pulsar events and to measure the masses of isolated pulsars with advanced optical facilities.

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**Table 1.** Summary of previous survey simulation results of FAST and SKA. Numbers in the parentheses represent known pulsars.  $l$ ,  $b$  and Dec are the longitude, latitude and declination in the galactic coordinate system, respectively.

Detectable pulsars		FAST <sup>c</sup>		SKA <sup>d</sup>	
	All Sky	$20^\circ < l < 90^\circ$ $ b  \leq 10^\circ$	$20^\circ < l < 90^\circ$ $ b  \leq 10^\circ$	$0^\circ < l < 85^\circ$ & $155^\circ < l < 360^\circ$ $ b  \leq 5^\circ$	Dec $< 50^\circ$
Normal pulsar	$\sim 30000^a$	$\sim 5700(352)$	$\sim 7000(418)$	$\sim 11000$	$\sim 14000$
Millisecond pulsar	$\sim 30000^b$	$\sim 550(14)$	$\sim 770(20)$	$\sim 4000$	$\sim 6000$

Note. – The results are from “a”: Lorimer (2006), “b”: Lyne et al. (1998), “c”: Smits (2009b), “d”: Smits (2009a).

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